



APPLIED
ACADEMICS

SRVIXSM White Paper

Executive Summary

The purpose of this document is to provide a detailed explanation of the methodology underlying the calculation of the Cboe Interest Rate Swap Volatility Index (“SRVIXSM”). While some of the salient details will be familiar to those with a working knowledge of the Cboe VIX Methodology, the reader is encouraged to be mindful of significant differences in the main index formula arising from the mechanics of interest rate swaptions. For those interested in rigorous theory, the Appendix carries discussions regarding important mathematical differences between pricing volatility in equity and interest rate derivatives markets.

Additional resources for SRVIX:

- [SRVIX Factsheet](#)
- [SRVIX FAQ](#)
- [Empirical Primer](#)

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Methodology Summary

Cboe Exchange, Inc. (Cboe), in its capacity as a reporting authority, calculates and disseminates the Cboe Interest Rate Swap Volatility Index (ticker: SRVIX) based on an adaptation of the methodology used to calculate the Cboe Volatility Index®, commonly known as the VIX® Index (ticker: VIX). SRVIX is a financial benchmark designed to be an up-to-the-minute market estimate of the expected volatility of forward ten-year swap rates, and is calculated by using swaption prices from top interdealer brokers. More specifically, SRVIX is intended to provide an instantaneous measure of how much the market expects forward ten-year swap rates to fluctuate in the one year from the time of each tick of SRVIX.

The SRVIX Methodology extracts information from prices of swaptions across multiple strikes referencing a particular currency / maturity / tenor combination, e.g. USD 1Y-10Y, and distills it down to one number that represents the consensus view on the corresponding forward swap rate volatility over the maturity period. OTC interest rate swap and swaption markets constitute an important market for hedging interest rate exposures and expressing views on monetary policy and other macroeconomic risk factors, and their prices contain salient information about the future direction and volatility of interest rates.

Similar to VIX, the Methodology produces an expected volatility number that does not depend on any particular options pricing model or strike. SRVIX levels can be interpreted as the fair market price of volatility in the sense that it is the square root of the strike of a variance swap on forward swap rates. ATM implied volatility, on the other hand, is a strike-dependent parameter tied to a specific model with no such natural interpretation.

SRVIX is quoted as a basis point interest rate volatility to keep consistent with the convention of OTC swaptions, which reference rates instead of prices; this is in contrast to the Cboe/CBOT 10-Year US Treasury Note Volatility Index (“TYVIX”), which is quoted as a percentage price volatility.

VIX and SRVIX are conceptually analogous indexes that reflect the level of uncertainty in US equity and interest rate swap markets, respectively, and SRVIX provides an easy way for market participants to track the level of uncertainty in USD interest rates, which has wide-reaching impact for many investors. The conceptual uniformity within the VIX family also allows investors to make apples-to-apples comparisons between SRVIX, VIX, and other indexes in the VIX family to monitor trends in relative levels of uncertainty between bonds, equities, and other asset classes.

Index Factsheet

Name	Cboe Interest Rate Swap Volatility Index
Ticker	SRVIX
Index Type	Fixed income volatility
Volatility unit	Basis point yield volatility
Index overview	The SRVIX Series tracks fluctuations in the uncertainty around USD interest rates by measuring the market’s expectation of how volatile 10-year forward swap rates will be over the next 1 year. It is part of the Fixed Income VIX family of forward-looking option-implied volatility indices.
Index publisher	Cboe Global Markets
Methodology outline	SRVIX calculations are based on an adaptation of the VIX methodology to options on interest rate swaptions. The formula aggregates prices of near- and next-term at-the-money (“ATM”) and out-of-the-money (OTM) payer and receiver swaptions traded over the counter (OTC), and produces an expected volatility number.
Input data	Top interdealer-brokers in the USD swaption market.
Calculation hours	8:30am to 3:15 pm Central time
Publication frequency	Intraday. Updated every 15 seconds.
Historical data	Available from June 2012
Data vendors	Visit here for vendor list

Index Formula

The SRVIX calculation is based on a simplified option-implied pricing formula for a hypothetical variance swap contract on forward swap rates:

$$SRVIX_t(M, T) = 100^2 \times \sqrt{\frac{2}{PVBP_t \times M} \left(\sum_{i: K_i < R_t} SW_t^R(K_i, M, T) \Delta K_i + \sum_{i: K_i \geq R_t} SW_t^P(K_i, M, T) \Delta K_i \right)}$$

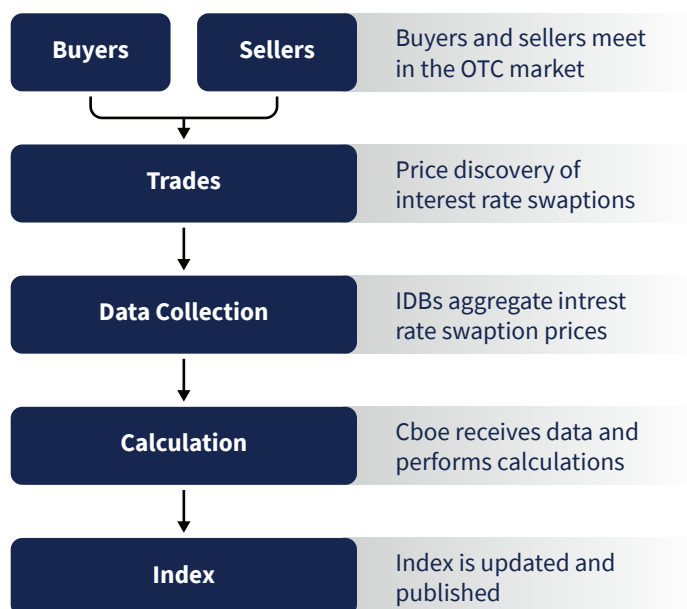
where variables are defined as

t	time of calculation	SW_t^R	price of a receiver swaption with strike K_i , maturity M , with underlying swap tenor T at time t
M	years to maturity of swaptions	SW_t^P	price of a payer swaption with strike K_i , maturity M , with underlying swap tenor T at time t
T	tenor of the underlying swap	K_i	i^{th} highest OTM strike available
R_t	M -year forward rate on a T -year swap at time t	ΔK_i	calculated as $K_i - K_{i-1}$ for the highest strike K_i ; $K_{i+1} - K_i$ for the lowest strike K_i ; and $(K_{i+1} - K_{i-1}) / 2$ otherwise
$PVBP_t$	price value of a basic point M -year forward rate on a T -year swap at time t		

and the quantity inside the square-root approximately represents the fair time t strike of a variance swap on R_t expiring at M , as discussed in the Appendix.

Input Data

SRVIX is fueled by multiple real-time feeds of ATM and skew implied lognormal volatility data from the top inter-dealer brokers (IDBs) in the OTC swaptions market. IDBs update their volatility cubes throughout each trading day based on the trading flows each observes between major sell-side dealers. As with most OTC markets, there is no centralized source of unique best bids and offers, and the more trade flow one sees, the more one's prices are likely to be representative of the overall market. Therefore, aggregating data from multiple IDBs, each of which aggregates information from the major dealers that further aggregate information from all of their buy-side clients, provides robustness in the price discovery process required for Cboe to calculate and disseminate SRVIX.



Implementation

SRVIX is an implementation of the theoretical formula above to calculate a constant rolling 1-year implied volatility of 1-year forward 10-year swap rates based on available OTC swaption prices. But it should be obvious to the reader how the calculation generalizes to other currency / maturity / tenor combinations.

Step 1: Choose swaption maturity and swap tenor

Unlike exchange-listed options, OTC swaption dealers quote prices for a constant set of maturity and tenor lengths, as opposed to fixed listing dates. Therefore, there is no need to choose more than one option maturity to interpolate or extrapolate to achieve a constant 1-year implied volatility from one day to the next as is done for VIX and TYVIX. Figure 1 provides an example of implied volatilities and premiums divided by PVBP for 1Y-10Y USD swaptions.

Figure 1: sample 1Y-10Y USD Swaption data

Offset	Strike (%)	Lognormal Imp Vol	BP Imp Vol	Receiver Prem/PVBP	Payer Prem/PVBP
-200	0.1319	41.4	88.26	0	200
-150	0.6319	41.21	87.86	0.02	150.02
-100	1.1319	41.03	87.47	1.67	101.67
-50	1.6319	40.84	87.07	11.61	61.61
-25	1.8819	40.74	86.85	21.38	46.38
ATM	2.1319	40.65	86.66	34.34	34.34
25	2.3819	40.55	86.45	50.09	25.09
50	2.6319	40.46	86.26	68.16	18.16
100	3.1319	40.27	85.85	109.32	9.32
150	3.6319	40.48	86.3	154.89	4.89
200	4.1319	40.79	86.96	202.63	2.63

Step 2: Choose swaption strikes

SRVIX uses the following set of strikes: ATM, +/- 25, 50, 100, 150, 200.

Step 3: Calculate SRVIX level

Payer and receiver swaption prices based on the Black (1976) formula are given by

$$SW_t^P(K, M, T) = PVBP_t \times [R_t \Phi(d_1) - K \Phi(d_2)]$$

and

$$SW_t^R(K, M, T) = PVBP_t \times [K \Phi(-d_2) - R_t \Phi(-d_1)]$$

for $d_1 = \frac{\ln(R_t / K) + (\sigma^2 / 2)M}{\sigma \sqrt{M}}$, $d_2 = d_1 - \sigma \sqrt{M}$ where σ is the lognormal volatility parameter, Φ is the standard cumulative

normal distribution function, and the rest of the variables are as defined in the previous section. Because the $PVBP$ in the denominator of the SRVIX formula cancels out the $PVBP$ in the Black (1976) formula, one only needs to calculate

$[R_t \Phi(d_1) - K \Phi(d_2)]$ for payers and $[K \Phi(-d_2) - R_t \Phi(-d_1)]$ for receivers, which equal the premium divided by $PVBP$.

Figure 2: Intermediate calculations for each strike

Offset (bps)	Strike (%)	Type	Prem / PVBP (bps)	ΔK_i (%)	Prem / PVBP x ΔK_i (% x bps)
-200	0.1319	Receiver	0	0.5	0
-150	0.6319	Receiver	0.02	0.5	0.01
-100	1.1319	Receiver	1.67	0.5	0.835
-50	1.6319	Receiver	11.61	0.375	4.35375
-25	1.8819	Receiver	21.38	0.25	5.345
ATM	2.1319	ATM	34.34	0.25	8.585
25	2.3819	Payer	25.09	0.25	6.2725
50	2.6319	Payer	18.16	0.375	6.81
100	3.1319	Payer	9.32	0.5	4.66
150	3.6319	Payer	4.89	0.5	2.445
200	4.1319	Payer	2.63	0.5	1.315

Figure 2 lists the relevant ATM and OTM swaption premiums divided by ΔK_i and the corresponding values of A_t and the corresponding values of ΔK_i . Multiplying the two row-wise and summing across all strikes yields 40.63 E-06, which leads to the index value

$$SRVIX_t(1,10) = 100^2 \times \sqrt{2 \times 40.63 e^{-6}} = 90.15 \text{ bps}$$

which is higher than the ATM implied basis point volatility of 86.66 bps, although this need not be the case depending on the shape of the skew.

Step 4: Aggregate across IDBs

Repeat Steps 1 – 3 based on pricing from each IDB, and take the median across all contributors.

Figure 3: SRVIX historical levels



Appendix: Theory and Implementation

A mathematically rigorous treatment of variance swaps on forward swap rates and the theoretically precise pricing formula can be found in Mele and Obayashi (2014, a). According to the theoretical variance pricing formula, the fair market value of a forward swap variance swap is the expectation of future realized variance taken under the so-called forward annuity probability, not under the risk-neutral probability, as in the standard equity variance case, which assumes constant interest rates (e.g., Cboe VIX). This feature of fair value arises from the fact that swaptions have two sources of randomness—the forward swap rate itself and the forward PVBP prevailing at expiry of the swaptions—and the obvious fact that interest rates cannot be taken to be constant in the context of interest rate volatility. While it may appear subtle, an understanding of this property is of fundamental importance when pricing derivatives that may become traded on the index. For practical purposes, a modification is made to the theoretical variance pricing formula to arrive at the SRVIX formula presented above. The mathematical derivation assumes the availability of swaption prices for a continuum of strikes ranging from zero to infinity. In practice, only finite sets of swaptions are quoted on OTC markets, which leads to the discrete strike-weighted sums in the SRVIX formula. This is an unavoidable reality of any options market, but it is straightforward to profile the approximation error by varying the range and increment of strikes used to calculate the index.

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Fixed Income VIX Family

- Cboe/CBOT 10-year U.S. Treasury Note Volatility IndexSM
- Cboe Interest Rate Swap Volatility Index (USD)
- Cboe/IHS Markit CDX North American Investment Grade Volatility Index
- Cboe/IHS Markit CDX North American High Yield Volatility Index
- Cboe/IHS Markit iTraxx Europe Main Volatility Index
- Cboe/IHS Markit iTraxx Europe Crossover Volatility Index
- S&P/JPX Japanese Government Bond Volatility Index[®]

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